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ADVANCEMENTS IN THE CLASSIFICATION OF TOPOLOGICAL PHASES OF MATTER

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ABSTRACT

Classifying matter into topological phases ushers in a new era in condensed matter physics, wherein phases are defined not by conventional local order factors but by global topological features. Topological insulators and topological superconductors are prime examples; the former includes conductive edge states shielded by time-reversal symmetry and the latter hosts Majorana modes that could be used in quantum computing. Furthermore, fractional quantum Hall states expand the quantum phase space by introducing fractional statistics and anyonic excitations. This categorization scheme has shown robust states that are resistant to some kinds of disorder and environmental fluctuations as a result of the creation of theoretical tools such as symmetry-protected phases and topological invariants, which are ideal for use in new technology. Novel topological phases are being identified in ongoing research, which opens up a wide panorama of possibilities for both basic science and sophisticated material applications.

Keywords: Topological phases, superconductors, fractional quantum, computing, Dimensional.

I. INTRODUCTION

By combining ideas from topology, quantum physics, and materials science, the categorization of topological phases of matter is a major step forward in our comprehension of condensed matter physics. Unlike conventional phases of matter (solid, liquid, gas), topological phases are defined by global qualities that are unaffected by smooth deformations, as opposed to local order parameters. This categorization has resulted in the identification of previously unknown matter states that defy the usual symmetries linked with phase transitions. Topologically protected states can provide robustness against certain sorts of disturbances, which has ramifications for quantum computing and has created routes for novel materials thanks to pioneering work in this field.

One group of topological phases that has received a lot of attention is topological insulators. While these materials act as insulators when in bulk, they can take on conducting properties when exposed to certain surfaces or edges. The existence of surface states protected by time-reversal symmetry is the defining feature of topological insulators. Such states are generated by the non-trivial topology of the band structure, which is typically shown by a "topological invariant," like the Z2 invariant. Theoretically, this material categorization lays the groundwork for comprehending spin transport and quantum Hall effects by linking certain electrical qualities to underlying topological features.

Topological superconductors are a kind of topological insulator that also displays surface or edge states but with superconducting characteristics. Exotic Majorana zero modes, which are their own antiparticles and have been suggested as potential candidates for fault-tolerant quantum computers, can be hosted in the edge states of topological superconductors. These modes are a result of the interaction between topology and superconductivity, and they open up new avenues for theoretical and applied study in quantum technology.

A further important class of topological phases are the fractional quantum Hall states. When electrons in two-dimensional systems are compressed into a fractional-excitation state by application of intense magnetic fields, these phases manifest. In contrast to the quantized Hall conductance in the integer quantum Hall effect, the fractional quantum Hall effect displays a rich topologically ordered structure encompassed by a collection of invariants. Anyonic excitations, which can exist in a superposition of states and exhibit fractional statistics, are indicated by these invariants, such as the filling fraction. Our knowledge of quantum statistical physics is expanded by this occurrence, which further muddies the boundaries between different sorts of particles.

These are only a few examples of the many possible ways topological phases might be categorized. Scientific investigations have uncovered an abundance of unusual phases in different systems and dimensions, such as higher-dimensional topological phases and those with crystalline symmetry. As an example, crystalline topological insulators integrate topological and crystalline symmetries in a novel way by incorporating the underlying lattice's symmetry. In this unique geometry-topology interaction, these systems display edge states that are shielded by both the band structure topology and the particular symmetries of the crystal lattice.

The categorization of topological phases has also been propelled by theoretical developments, backed by experimental findings. To classify and characterize these exotic states, tools like symmetry-protected topological phases and topological field theories have been created. Discovering topological invariants provides a window into the stability of topological matter under real-world conditions by allowing researchers to forecast phase transitions and measure the robustness of these phases against disturbances.

Furthermore, future technical applications are affected by the continuous investigation of topological phases. Topological phases have great potential for use in quantum computing, secure data transport, and new kinds of electronic devices due to their resistance to disorder and disturbances. Discoveries of novel compounds with desirable topological traits have been spurred by the pursuit of materials showing these qualities, which in turn has increased interest in synthesis techniques and material characterization.

A new way of thinking about condensed matter physics has emerged with the categorization of topological phases of matter. This area is constantly developing, providing fresh understanding of matter's basic structure and potential uses that can revolutionize technology through illuminating the deep relationships between topology and physical characteristics. Theoretical frameworks and practical advances in materials science will be enhanced as research continues to reveal more detailed links between topology and quantum mechanics, which will likely lead to broader implications of these categories.

II. REVIEW OF LITERATURE

Zhang, Lin et al., (2018) the study of the topological phase of matter is now standard practice in the field of condensed matter physics. Even if there is yet no clear solution in theory and practice, topological phase classification, synthesis, and detection have recently piqued the interest of academics. In order to find integer-classified equilibrium topological quantum phases, we suggest a universal non-equilibrium characterization and high-precision dynamical methods. Fundamental theorems are employed in dynamical classification theory. We demonstrate first that a generic multiband's d-dimensional (dD) gapped topological phase may be reduced to a (d-1)D invariant defined on band inversion surfaces (BISs), leading to a bulk-surface duality that makes topological characterization easier. In addition, we show that distinct topological patterns on BISs are produced by (pseudo) spin dynamics in quenching over phase boundaries. These patterns are explained by the post-quench bulk topology and disclose a dynamical bulk-surface link. A dynamical topological invariant derived from an emerging dynamical spin-texture field on the BISs is used to categorize the topological phase. New topological phase detection techniques show great feasibility when applied to quenching tests on practical models. It opens a new way to use non-equilibrium quantum dynamics for topological phase detection and classification.

Chiu, Ching-Kai et al., (2016) There has been a lot of study on topological materials because they display unique physical phenomena that could have applications in quantum information technology and inventive technologies. The nontrivial topology of the bulk wave functions gives rise to protected gapless surface states in topological materials. Topological quantum matter is introduced in this review through the use of classification systems. We categorize topological materials that are either totally gapped or entirely gapless according to global and spatial symmetries such as reflection and time-reversal. Furthermore, topological defect-localized gapless modes are categorized. These systems are categorized using Homotopy groups, Clifford algebras, K-theory, and non-linear sigma models that depict Ander-son (de-)localization either on the surface or within a material defect. A unified and thorough grasp of the subject is achieved by comparing and contrasting modern experimental results with theoretical model systems and their topological invariants. Although we touch on unresolved challenges and new findings regarding interactive system topological classifications, the main emphasis of this work is on the topological properties of noninteracting or mean-field Hamiltonians.

Shen, Shun-Qing. (2014) New condensed matter materials, such as topological insulators and superconductors, have been found. Boundary states are properties of materials, whereas topological phases are attributes of their global quantum states. Condensed matter science has never seen these phases before. A historical development and the family of condensed matter phases are summarized in this article.

Mesaros, Andrej & Ran, Ying. (2012) One novel quantum phase of matter that has recently attracted attention are symmetry-protected topological states, such as topological insulators. Interactions are categorized by group cohomology. These stages are characterized by localized entanglement and the absence of bulk topological order. There is less clarity in identifying quantum phases of matter with global symmetries when there is long-range entangled topological order. We classify bosonic gapped quantum phases according to whether they have on-site global symmetry and long-range entanglement or not. In 2+1 dimensions, quantum phases with a topologically ordered finite gauge group GG and a global symmetry group SG are classified by the cohomology group H^3. It is common practice to use H^{d+1}(SGxGG,U(1)) for classifying quantum phases in d+1 dimensions. Although our understanding of its completeness is limited, we provide a locally bosonic model that is both solvable and exhibits emergent topological order for every class in our classification. The topological order in models lacking global symmetry is characterized by generic Dijkgraaf-Witten discrete gauge theories. In the absence of topological order, our models are uniquely amenable to solution for topological phases that are protected by symmetry. Our models characterize topological phases that are enriched with symmetry when both global symmetry and topological order are present. The observable characteristics of these topological phases that are enhanced by symmetry, as well as extensions of our classification that go beyond the projective symmetry group classification, are being studied.

III. TOPOLOGICAL ORDER

A. Moving beyond Landau's model

Parameters of order, symmetry breakdown the topo-logical phases are so called because they are dependent on the material's topology and are resistant to local disturbances. A few examples of topological quantities are entanglement, non-abelian geometric phase, resilient ground-state degeneracy, and others.

To comprehend the formation of topological phases, it is necessary to first define a few essential terms. There are two ways to describe the quantum phases of matter: the Hamiltonian and the state. To begin, we have the Hamiltonian method.

B. Hamiltonian phase transitions

Take into account a collection of Hamiltonians that correspond to the category of systems being investigated, such as fermionic/bosonic Hamiltonians or Hamiltonians with a given symmetry. This set allows us to define phase transition.

An explanation of quantum phase transitions Assume that ψ0(g) is the ground-state of a Hamiltonian family H(g). If the singularity of an observable O at gc indicates a phase transition, then a phase transition occurs at gc.

According to the classical definition, this describes a phase transition. Since it is necessary to examine all observables simultaneously to ascertain whether a phase transition occurs, it is not practical for phase classification. The following theorem provides a more convenient characterization:

Hypothesis 1 Phase transitions can only take place when the energy gap of H(g) closes at gc.

This theorem is shown in Figure 1.

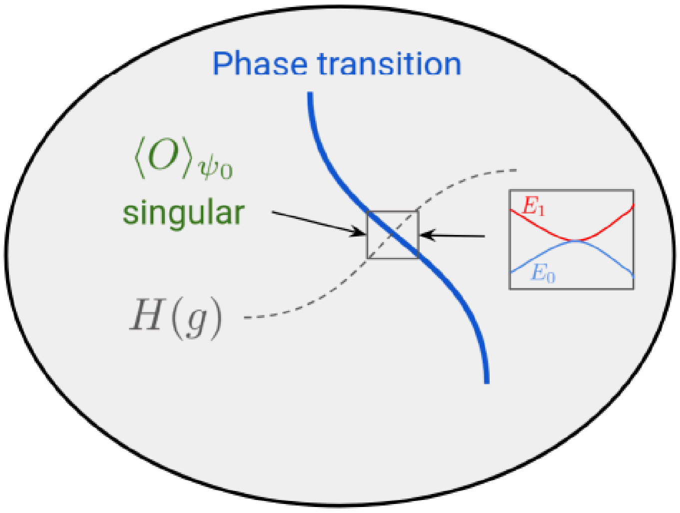


Figure 1 Diagram of a phase transition. The Hamiltonians' H is gray. If the energy gap closes or an observable becomes singular, a route of Hamiltonians H(g) ∈ H will experience a phase transition.

In quantum mechanics, a phase transition is not necessary for the connection of some classes of Hamiltonians. Quantum phases in Z2-symmetric Hamiltonians include the symmetric and symmetry-breaking stages of the Ising model (see Figure 2). Keep in mind that different sets result in distinct stages. Avoiding inevitable phase changes within requires thinking about external pathways.

The collection G of all gapped Hamiltonians (those whose gap does not close in the thermodynamics limit) is a large class of Hamiltonians that gives rise to topological order. Topological order 1 can be defined for this set:

The definition of topological order is 2. If the ground-state degeneracy of a G phase is stable in the face of local perturbations, we say that the phase has topological order.

The quantum error-correcting toric code exhibits topological order. With appropriate local noise shielding, the logical qubits 00, 01, 10, and 11 are encoded in the 4-dimensional stable ground space of a given Hamiltonian.

Using states instead of Hamiltonians to represent phase transitions allows us to classify all topologically ordered phases.

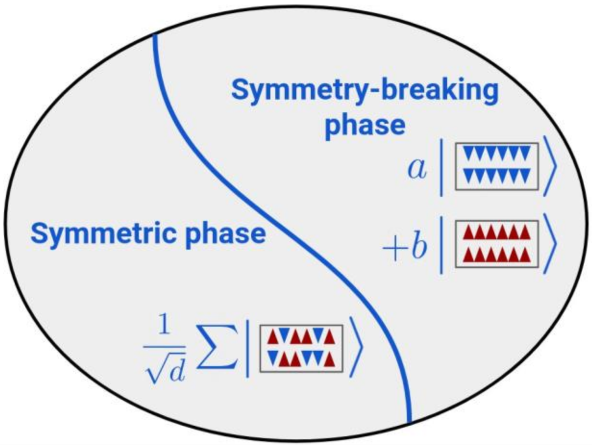


Figure 2 The two stages of the Ising model exhibit distinct symmetry properties, ground state degeneracies, and a transition-induced gap closure.

C. Changes in phase from quantum states

For a given set, we can say that all ground states are Hamiltonian. If the gap between the ground states of two Hamiltonians is not filled in by a path H(g), we say that the two states ψ(0) and ψ(1) are in the same phase. While this definition is simple, it is not applicable to states because it includes Hamiltonians. Phase characterization from a state perspective is enhanced by the following theorem when G=:

Second Hypothesis The only way to link two states that are in the same phase is by a constant-depth quantum circuit, also known as a local unitary evolution.

The tools for identifying phases of matter are expanded through this characterization, which employs quantum information. Because of this, the following consequence follows:

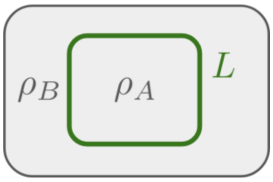
Next Steps A topologically ordered phase of matter in G comprises highly entangled states, while a trivially ordered phase has the product state.

A constant-depth local circuit cannot yield highly-entangled states in quantum information theory; thus, a minimum of two steps are required. The topological order as defined by the state is identical to the one defined by the Hamiltonian. The entanglement of topological orders is long-range, whereas that of trivial orders is short-range.

A topological order can exist in multiple phases, but a trivial order can only have one (the product state). Among our categorization goals is the compilation of a complete set of topologically-ordered G phases along a given dimension.

D. Characteristics of maximally ordered sets

Long-range entanglement and stable ground-state degeneracy characterize topologi-cal order. This final characteristic was measured by Kitaev and Preskill in 2005 when they found topological entanglement entropy. The two regions A and B that make up a 2D quantum state ψ are separated by a size L barrier, and they define partial states ρA and ρB, respectively.



The area law requires that the Von Neumann entropy of a ground state ψ, which is defined as S(ρA) = Tr[ρA log(ρA)], be proportional to L rather than the volume of A. It has been demonstrated by several specific cases, such frustration-free Hamiltonians, that the area law is applicable to any gapped ground state. Topological order allows one to demonstrate an alternate area law:

S(ρA) a αL − γ

The toric code's offset γ, which is synonymous with log(2), defines topological entanglement. Typically, γ is not zero for topologically ordered states.

Scattering quantum Hall states, chiral spin liquids, and the toric code are all examples of topologically ordered states.

E. Topological phases that are protected by symmetry

Phases of gapped Hamiltonians (G) that lack symmetry have been our whole focus up to this point. As a result, we are understandably curious in the consequences of applying symmetries to the Hamiltonians in question. Our symmetry can partition the insignificant order into multiple stages, as seen in Figure 3. Symmetry-Protected Topological (SPT) phases are more complex and difficult to categorize than topological order, despite being defined by short-range entanglement. Two examples of SPT phases are topological insulators and superconductors and the integer quantum Hall effect. A symmetry-breaking phase can also arise in our Hamiltonian due to its symmetries. The preservation of symmetry is what differentiates SPT phases from symmetry-breaking phases in the literature.

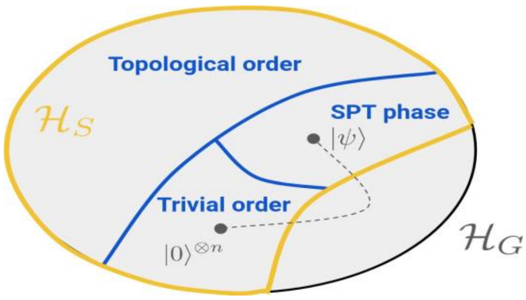


Figure 3 A different route (dashed line) can always be taken in the set HG of gapped Hamiltonians without symmetry requirements to prevent phase transitions of the trivial order.

When taking into account a limited set of symmetric Hamiltonians HS, this phase transition can produce symmetry-protected phases with short-range entanglement.

Furthermore, new topological order phases can be introduced by symmetry restrictions. When they do not break symmetry, they are called Symmetry-Enriched Topological (SET) phases. Topological Mott insulators and fractionalized topological insulators are examples of these phases.

This first layer of classification is summarized in Figure 4.

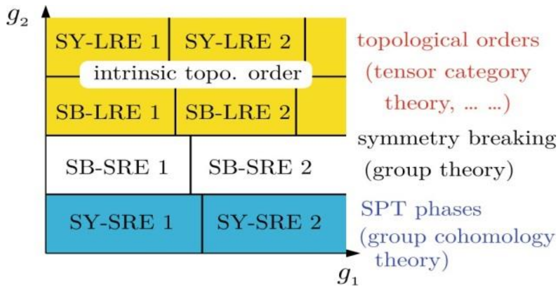


Figure 4 First layer of classification of topological phases, from

Differentiating between in-trinsic topological order and short-range entangled (SRE) states is the initial step. Then, inside each region, various phases can emerge through symmetry-breaking (SB) or symmetry-preserving (SPT) or symmetry-setting (SET) phases.

IV. TOPOLOGICAL PHASE CLASSIFICATION

We proved that topological matter goes through four distinct phases: trivial, topologically ordered, symmetry-enriched, symmetry-protected. The Hamiltonian and ground-state symmetry groups (GH, Gψ) allow us to categorize symmetry-breaking phases; we aim to extend this categorization to all other phases. Next, we will examine systems with bosons or fermions that do not interact, systems with one dimension (complete classification), and systems with more than one dimension (partial classification).

Systems that do not interact

A fermionic or bosonic Hamiltonian system without interactions is the first completely classified system. There are only ten global symmetries for free fermions and bosons, and they are chiral, charge conjugation, and time-reversal. Using the ten-fold classification of free bosons/fermions, all SPT phases of topological insulators and superconductors may be described using K-theory methods. You can see this categorization in Figure 5.

	Symmetry			Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Figure 5 Differencing topological insulators and superconductors without interactions into ten distinct classes. Specifically, T stands for time-reversal, C for charge-conjugation, and S for chiral symmetry. We have S = T • C where T 2 is an element of the set {0, 1, −1}, and C2 is equivalent. In

each dimension, Z represents an infinite number of phases, whereas Z2 signifies two.

Systems with only one dimension

In 2011, two separate teams independently categorized systems with only one dimension. Included in this categorization are two noteworthy discoveries:

1. All ground states in 1D have short-range entanglement, hence there is no topological order.
2. The SPT phases can be determined using the formula (GH, Gψ, ω), where ω is a projective representation of Gψ in the second cohomology group H2(Gψ, U (1)). The bosonic system symmetries and their labels are listed in Figure 6.

Symmetry	No. or Label of different phases
None	1
On-site symmetry of group G (*)	$\omega \in H^2(G, U(1))$
Time Reversal (TR)	2
Translational Invariance (TI)	1
TI + On-site linear symmetry of group G	$\omega \in H^2(G, U(1))$ and $\alpha(G)$
TI + On-site projective symmetry of group G	0
TI + Parity	4
TI + TR	2 if $T^2 = I$ 0 if $T^2 = -I$

Figure 6 The classification of 1D boson/spin systems

Two primary theories were utilized by both authors to arrive at those results: 2) By eliminating degrees of freedom, generalized local unitary evolution maintains phase, and all one-dimensional ground-states satisfy the area law and may be written as MPS. Finding a renor-malization technique that uses those basic principles to push any MPS to a product state proves point 1.

Towards higher dimensions

At long last, there has been progress in higher-dimensional system classification. 2D systems that exhibit topological order, unique ground state degeneracy, or PEPS ground state (a 2D generalization of MPS) can be classified using projective representations, just like 1D systems. 2D mathematical formalism typically requires more work. It is possible to identify fermionic systems with fusion categories and 2D bosonic systems with modular tensor categories.

Some point-like excitations in three dimensions can be fermions, while others can be completely bosonic. Symmetries in these cases can also exhibit properties of topological insulators and superconductors.

V. CONCLUSION

In conclusion, the classification of topological phases of matter has revolutionized condensed matter physics by introducing topology to physical attributes. Topological phases, which are determined by global properties rather than local order factors, are resilient, symmetry-protected, and promising for advanced technological applications. This new categorization covers topological insulators, superconductors, and fractional quantum Hall states, which have edge or surface phenomena that contradict standard models. Theoretical frameworks and experimental advances have expanded the scope of known topological phases and shown their stability against certain disorder and perturbations, which could be useful in developing quantum computing and secure information technologies. Topological phases' environmental resistance boosts their potential in stable, precise applications. New phases are predicted to be uncovered as study progresses, expanding the classification and enhancing our understanding of quantum mechanics. Topological phases are crucial to understanding matter and developing innovative technologies, as this discipline combines fundamental research with prospective innovation.

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